Random wave runup at a plane beach

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Introduction

Accurate estimation of irregular wave runup at sloping beaches is important in assessments of coastal erosion and flood risk. We define runup as the vertical elevation of the shoreline position, driven by swash oscillations about the wave setup level. To date there have been rather few investigations into irregular wave runup statistics, and these have assumed linear, Gaussian behaviour. This presentation will describe use of a 1-D hybrid Boussinesq-NLSW model to produce statistical information on the runup of irregular waves, arising from a random sea state offshore of the beach. A validated technique will be presented for simulating long-duration irregular runup time series whereby the incident waves are generated using second-order accurate paddle motions with reflected waves effectively absorbed before they reach the paddle.

Numerical Model

The numerical wave flume solves a weakly dispersive, weakly nonlinear version of the Boussinesq-type equations (see Madsen & Sørensen, 1992) using fourth-order finite differences for non-breaking waves, and the nonlinear shallow water (NLSW) equations using a second-order MUSCL-Hancock HLLC Godunov-type finite volume scheme. A local mapping is used to facilitate the paddle-type wave generator. The switch between the Boussinesq and NLSW solvers is triggered by a wave breaking criterion set according to the local water free surface slope, with dispersive terms turned off gradually. Flux gradient and source term balancing of the NLSW solver is achieved by recasting the equations in deviatoric form with wetting and drying modelled using an algorithm developed by Liang & Borthwick (2009). Full details of the model are given by Orszaghova *et al.* (2012, 2014). To generate the irregular waves, we use the second-order wave generation theory by Schäffer (1996) without first-order evanescent contributions to second-order error waves. A generating-absorbing sponge layer proposed by Zhang *et al.* (2014) is used to achieve effective wave absorption over the range of the incident wave frequency spectrum. Irregular wave runup is modelled by running concurrently (1) a simulation of incident irregular waves over a flat bed, in the absence of a beach, which are absorbed in a sponge layer, with (2) a simulation of the irregular waves propagating across the generating-absorbing sponge layer (with the solution from (1) imposed on the sponge layer) and up the plane beach. Fitzgerald *et al.* (2015) provide further details of the methodology and its verification tests.

Results

Irregular waves were simulated over a 1/10 reflective, 1/20 intermediate, and 1/40 dissipative beach slopes with corresponding domain lengths of 18, 24, and 32 m. The input spectrum was Pierson-Moskowitz with peak frequency 0.464 Hz. The still water depth in the flat bed region between the paddle and beach toe was 0.5 m. The spectral zero-crossing period was Tz = 1.75 s, with 280 waves equivalent roughly to a return period. The significant wave height was 0.1 m. For each slope, sixty wave records were acquired, each of 491.52 s repeat period. Figs. 1and 2 display part of the free surface elevation (at beach toe) and shoreline elevation time histories for the 1/20 beach slope (with Froude scaling for 20 m offshore depth), the red dashed lines are for linear paddle signal motion and the black solid for linear + 2nd order. Fig. 3 presents histograms of frequency of occurrence against maximum runup crests that were determined from the full repeat duration shoreline elevation records obtained for each beach slope, and include the wave setup effect. A secondary peak (indicated by the red arrow) can be discerned in the upper tail of each histogram. Its presence indicates that there may be an upper limit to the maximum runup crest elevation (as has been observed from the analysis of swash spectra on beaches of mild slope). It can also be seen that the width of the probability distribution decreases progressively as the beach slope reduces (noting that the plots have different scales). There are more runup events / time on the steeper slopes, meaning that the distributions are not exactly equivalent statistically.

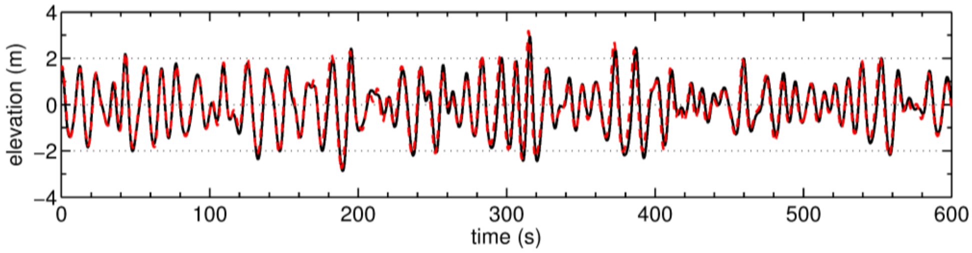

Fig. 1 Free surface elevation at beach toe at 1/20 beach slope (after Froude scaling for 20 m offshore depth).

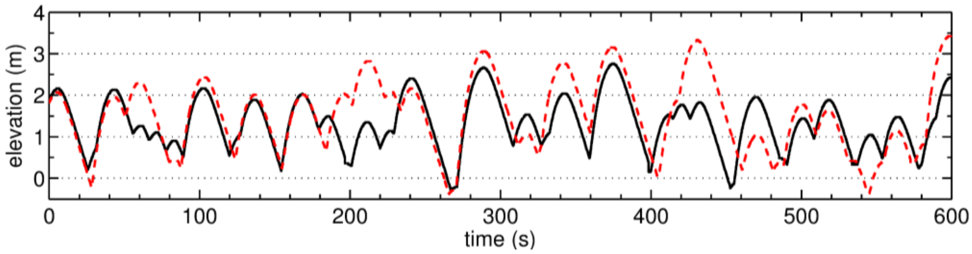

Fig. 2 Shoreline elevation time history at 1/20 beach slope (after Froude scaling for 20 m offshore depth).

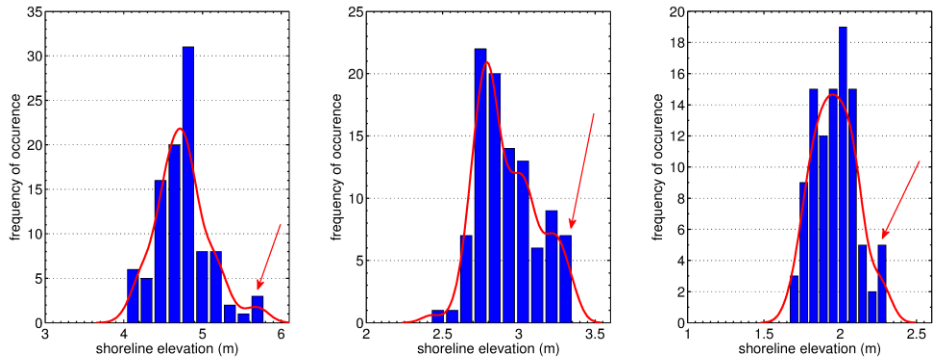

Fig. 3 Max. runup crest values on beach slopes of (a) 1/10, (b) 1/20, and (c) 1/40. Arrows = ensemble maxima.

Conclusions

A validated methodology has been described for the simulation of irregular wave runup at beaches in a 1-D flume, with wave generation according to second-order wavemaker theory, and reflected waves properly absorbed by a sponge layer before reaching the offshore boundary. The method has been used to obtain random wave runup statistics at examples of reflective, intermediate, and dissipative beaches.

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